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CONTROL OF A 36 MODE HYBRID VEHICLE WITH DRIVER OPTION SELECTION - INCORPORATING URBAN, SUBURBAN, AND HIGHWAY DRIVING

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ABSTRACT

There is currently a high level of uncertainty surrounding the evolution of personal transportation. A variety of new types of vehicle powertrains have been proposed or implemented, including alternative fuels, hybrid electric vehicles, and fully electric vehicles. It is also possible, as shown by Mechtenberg [1], to combine multiple fuels and batteries to design this 36 mode hybrid vehicle. The hybrid vehicle presented here features multiple modes of operation with a wide range of possible combinations of fuel and battery usage. While the many degrees of freedom offered by this hybrid vehicle design present an opportunity to operate under a variety of different conditions, it also presents a control challenge, as the vehicle's control system must decide how best to use the various modes available, given the driver's optional selection and the current status of the vehicle. In this paper, we discuss the various modes of operation, degree of driver involvement in their selection, and automatic switching between various options. The optimal control is found for various different driving cycles, based on the objective of maximizing the efficiency of the powertrain, and it is shown that this type of hybrid vehicle can operate efficiently under a variety of different scenarios. This model is built upon Wagner and Papalambros' engine optimization [2] and Ahn's continuously variable transmission model [3].

1 INTRODUCTION

In the early development of the automobile, there were many different types of powertrain configurations and drive systems, ranging from the early Otto cycle engines powered by gasoline, to diesel engines, to steam-powered vehicles such as the Stanley Steamer. While the Otto cycle became the dominant technology for much of the automobile's history, there are currently many research efforts focused on new types of powertrains and innovative drive technologies for the automobile. These primarily include purely electric vehicles, hybrid electric vehicles, and engines using alternative fuels. In this environment, a flexible drive train may be attractive, since it is not yet known which type of technology will emerge as the dominant paradigm for automotive design.

Flexibility, however, presents challenges. If a vehicle offers multiple modes of operation, then some means of choosing the option to be used must be found. This decision can be partly based on driver input, since the driver presumably knows how far he or she plans to travel and on what types of roads, but it also must be based on the vehicle's current state, since a driver cannot be expected to continuously, consciously monitor conditions such as battery SOC. In this paper, such a flexible powertrain is described, and its control strategy is presented. It is flexible enough to transition from one so-called locked-in technology to another locked-in technology.

We narrowed down the options based on the characteristics of drivers in urban, suburban, and highway driving. This includes 4 key alternative fuels for an internal combustion engine: gasoline, ethanol, natural gas, and hydrogen. Due to the medium and high power distinctions in modes of operation needed, cylin-

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der deactivation was included. For plug-in capability and low power, an electric powertrain was considered with two battery chemistries: low power with high energy density as well as high power with low energy density. It is noted that an ultra capacitor could have been chosen instead of battery chemistry with high power and low energy density.

The paper is organized as follows. Section 2 presents a brief review of literature in the areas of traditional HEVs and cylinder deactivation. In Section 3, we describe the hybrid vehicle powertrain and its driver-selected modes of operation. Section 4 presents the control optimization problem, with results given in Section 5. Concluding remarks are given in Section 6.

2 BACKGROUND

There is a large and rapidly growing body of work on hybridization, powertrain design, and fuel economy. Here, we present a brief review of two of the most relevant areas of literature to this paper, namely, the most common types of hybrid electric vehicles (HEVs) and the literature on cylinder deactivation. The conclusion of this brief literature review is that the number of hybrid vehicle permutations possible is vast, complex, and continues to grow.

2.1 Hybrid Electric Vehicles (HEVs)

Hybridization has been widely demonstrated as a method to improve fuel economy by reducing idling loses, recapturing braking energy, and reducing transient loses (e.g., [4–6]). Traditional operation of a spark-ignition engine at light loads involves higher levels of friction. However, the electric motor is more efficient at light loads. In addition, prior to adding an electric motor into the powertrain, braking energy was lost to heat, but now the electric motor can operate to charge the battery. Finally, some drivers tend to accelerate and decelerate often and for mild acceleration, the electric motor can operate for coasting speeds on the highway and mild transients; thereby, leaving the engine either off or in its efficient operating point.

The literature on hybrid electric vehicles, both plug-in (PHEV) and “traditional” HEVs, is vast, particularly in recent years, and has been the subject of numerous research efforts and review papers, e.g. [7, 8]. These works show that hybridization has a great deal of potential; however, like all technologies, it presents some challenges and disadvantages. Some drawbacks of hybridization stems from the advance controls needed, the addition of weight, and their interdependence on driving cycles (or how fuel economy depends on user). However, current use of hybridization illustrates that researchers and manufactures believe enough in the positive benefits to put this technology into production automobiles as well as future plans. In addition to these, combining cylinder deactivation with electric motor hybridization has also been manufactured in the Honda Accord 3.0 L V3-6 [9]. Adding two fuel lines into a cylinder deactivation electric hybrid engine offers an additional degree of freedom.

2.2 Cylinder Deactivation

Cylinder deactivation has been shown to be effective at improving fuel economy of vehicles; it accomplishes this by reducing the pumping losses of an engine (e.g., [10–12]). When a spark-ignition engine is operated at light loads, airflow is typically throttled to ensure that stoichiometric conditions are maintained in each cylinder. If there are fewer cylinders used at these light loads, then less throttling is required for the same power output. Using fewer cylinders may also improve the combustion performance and thermal efficiency since the cylinders will have a higher compression ratio, faster burn rate, and lower relative heat losses [11, 13]. In addition, there may be smaller frictional losses from fewer, more highly loaded cylinders than there would be from a larger number of lightly loaded cylinders [13].

As with every technology, cylinder deactivation presents some challenges and disadvantages; one of these challenges is the effect of a step change on the system. This step change in output of the engine is a result of switching cylinders on and off, a fundamentally discrete process. While this reduces the frequency of torque pulsations from firing of the cylinders, it can increase the amplitude of these pulsations at the crankshaft, and therefore one disadvantage is traded against another, presenting challenges in the management of noise, vibration, and harshness (NVH). The engine is also subject to asymmetrical loads. In addition, complex controls may be required in order to effectively manage the deactivation of the system and make proper use of it, and this may limit the practicality of the technology. However, cylinder deactivation presents enough benefits that it has been incorporated into some production automobiles, as listed below.

1916 - Enger ”Twin-Unit Twelve” V12-6 [14]
1981 - Cadillac Eldorado V8-6-4
1983 - Mitsubishi ORION-MD I4-2 [15]
1993 - Mitsubishi MIVEC I4-2 [16]
1998 - Mercedes 5.0L V8-4 and 6.0L V12-6 S-Class [17]
2004 - DaimlerChrysler 5.7L V8-4 HEMI [5] 2005 - Honda Accord 3.0 L V6-3 [9]

It is generally accepted, for any technology that reduces pumping losses, that heavier vehicles (i.e., those that have a higher ratio of engine size to vehicle mass) show the most advantages from implementation of the technology [13]. The combination of vehicle weight, engine size, and the gear ratio of the transmission has a major influence on the engine operation when cylinder deactivation is used [11], and thus a large impact on the fuel efficiency of the vehicle. It has been suggested that fuel consumption can be reduced from 3-6% by using cylinder deactivation (2002 National Research Council Report, as cited in [18]). Other researchers have calculated that 2.4-5.0% of fuel energy used in the EPA combined cycle test goes to pumping losses [13]; if the indicated efficiency is 37%, the elimination of these losses could yield 6.5-13.5% reduction in fuel consumption. Other benefits from cylinder deactivation have been shown in [19], which showed the potential for a 7-14% improvement depending on which drive cycle was used, and in [20], which showed a 23% increase in fuel economy over a conventional gasoline engine. A vehicle incorporating cylinder deactivation, then, has the potential to operate more efficiently under a wider range of conditions than a vehicle with a traditional engine that does not incorporate this feature.

3 CONFIGURATION AND DRIVER-SELECTED MODES OF THE HYBRID VEHICLE

3.1 Powertrain Configuration

The configuration of the hybrid vehicle's powertrain is shown in Figure 1. This powertrain utilizes cyilnder deactivation in order to provide the capability to use multiple fuels, with a split tank to provide the appropriate fuel to a given set of cylinders. One set of cylinders is designed for homogenous charge compression ignition cycle and assumed to run on the traditional fossil fuel, i.e., gasoline. The second set of cylinders is designed to use the Otto cycle, but the fuels modeled are alternative fuels. The three alternative fuels modeled are methane, ethanol, and hydrogen. Since these fuels have different combustion characteristics, the optimal dimensions of the cylinders are different, with the details of that optimization beyond the scope of this paper. The system also features two motor/generator units, to allow for more degrees of freedom in charging and discharging the batteries. Having two battery banks allows for the benefits of a small capacity battery for shallow charging cycles around (50% - HEV) and another for deep charging cycles due to plugging in (20 to 80% - EV and PHEV).

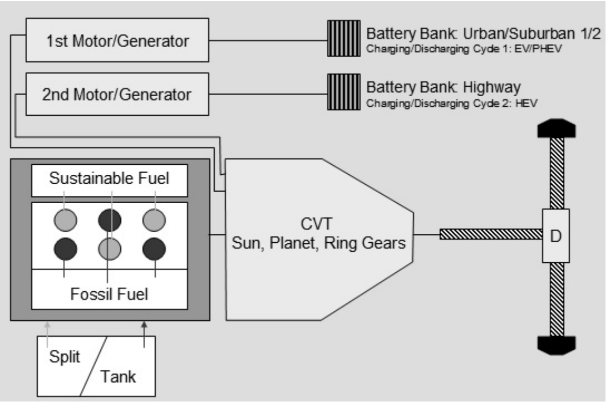


FIGURE 1. Schematic of the Hybrid Vehicle Drivetrain Configuration

All of the various powertrain options are connected to the drive wheels through a typical continuously variable transmission (CVT).

3.2 Driver-selected options and modes of operation

The various ways in which these options are combined is described by a six-digit binary coding scheme. In this scheme, shown in Figure 2, the first two digits indicate whether the first motor/generator is being operated as a motor (10), as a generator (01), or is turned off (00). Similarly, the third and fourth digits are used to characterize the second motor/generator. The final two digits are used to indicate the usage of the two sets of engine cylinders.

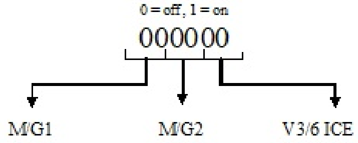


FIGURE 2. Coding Scheme for Modes of Operation

Mathematically, there are 64 possible permutations in this coding scheme; however, some of these are not physically possible, such as those that indicate that the motor/generator is running both as a motor and as a generator. Physically, 36 modes of operation are possible. Given the large degrees of freedom over a specific driving cycle, this coding scheme greatly enhances the functionality and more importantly speed of the control algorithm. However, the driver does not need to choose among these modes; he or she is given a set of four dashboard options, with switching among these 36 modes controlled by the vehicle's state and by the dashboard option selected.

The dashboard options are as follows:

1. Urban: all electric mode (EV) with limited range (10-30 miles) and plugging into the grid capabilities with deep cycle battery with very low acceleration capabilities and low maximum top speed capability
2. Suburban 1: plug-in hybrid with limited range (60-150 miles), but alternative fuel in otto cycle and medium acceleration capabilities (PHEV-O.AF)
3. Suburban 2: plug-in hybrid with switch to traditional hybrid with two fuel lines so highest acceleration capabilities (PHEV-O.AF)
4. Highway: traditional hybrid with both fuel lines but only uses shallow cycle battery for temporary acceleration (HEV-O.AF)

As the vehicle proceeds through a driving cycle, the choice of which options are turned on or off is made according to the strategy shown in Figure 3.

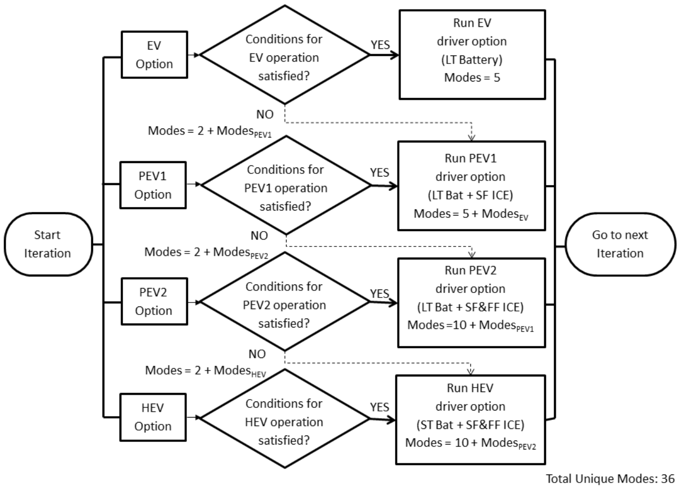


FIGURE 3. Power Management Strategy

The engine and generator/motor speeds that were used at each time step throughout the driving cycle were determined by optimization, with the control optimization problem defined in the next section.

4 CONTROL OPTIMIZATION FORMULATION

In this optimization, the objective function is to maximize efficiency (or, equivalently, to minimize inefficiency) of the system over a driving cycle. While hybrid vehicle optimization often is performed with an objective function to minimize fuel consumption, an objective based on efficiency has been used by some researchers (e.g., [21, 22]), and the efficiency-based objective offers the advantage that an efficiency can be easily defined for any energy system. This eliminates the need to define an equivalent fuel consumption for items that do not consume fuel, such as batteries. For each set of results, the fuel economy can also be calculated, and these results are reported in Section 5 of this paper.

The variables for this optimization are the speeds of the engines and motor/generators at each point along the driving cycle, with the driving cycle used being different based on the dashboard option chosen. The drive cycle used for each dashboard option is listed in Table 1. The objective function, then, is given as:

$$\min_{N_1, N_2, N_3, N_4} \left(1 - \prod_{i=1}^{n} \eta_i\right) \tag{1}$$

where N_1 is the speed of motor/generator 1, N_2 is the speed of motor/generator 2, N_3 is the speed of the engine using alternative fuel, and N_4 is the speed of the engine using fossil fuel; n is the number of components running at a particular time; and η_i is the efficiency of a given component. Given the speed of each component, it is possible to use the model in [2] to calculate all of the quantities neceesary to determine efficiency. Examples of the specific objective functions are given, for five of the different mode options discussed in Section 3.2, in Table 2.

TABLE 1. Drive Cycles Used for Dashboard Options

Dashboard Option	Driving Cycle
Urban	Japan 10-15
Suburban 1	NEDC
Suburban 2	50/50 UDDS and HWFET
Highway	HWFET

Once the driving cycle is known, the power requirement can be calculated based on the vehicle mass, velocity, acceleration, rolling and air drag resistance, and power required for accessories. The power requirement at a given point on the driving cycle, then, is given by

$$P_i = m v_i a_i + C_r m g v_i + \frac{C_r}{100} m g v_i^2 + \frac{1}{2} \rho C_d A v_i^3 + P_{accessories} \tag{2}$$

where the mass $m = 1800$ kg, the coefficient of rolling resistance $C_r = 0.01$, the drag coefficient $C_d = 0.50$, the area $A = 5.65\text{m}^2$, and $P_{accessories} = 0.75$ kW. The parameter ρ denotes the air density, and g is the gravitational acceleration.

It is assumed in the optimization that the full power requirement is always met. In addition, constraints are introduced to ensure that the engine and motor/generator speeds never exceed maximum safe limits, that the engine is never run in reverse (i.e., has a negative speed), and that the state of charge (SOC) of each battery remains within preset limits, as indicated in Eq. (3) - (8).

$$0 \leq N_1 \leq N_{1_{max}} \tag{3}$$

$$0 \leq N_2 \leq N_{2_{max}} \tag{4}$$

$$|N_3| \leq N_{3_{max}} \tag{5}$$

$$|N_4| \leq N_{4_{max}} \tag{6}$$

$$0.20 \leq SOC_1 \leq 0.80 \tag{7}$$

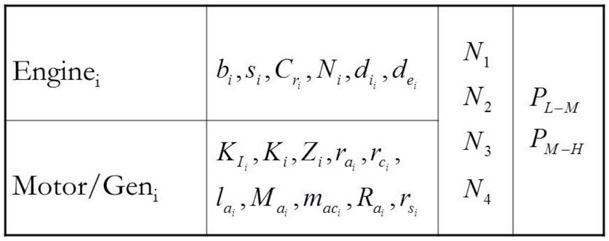
$$0.40 \leq SOC_2 \leq 0.60 \tag{8}$$

In addition, there is a constraint based on the Mach speed, which is typically active, and a constraint on the breakdown torque.

TABLE 2. Objective Functions for 5 Sample Mode Configurations

Mode Configuration	Objective Function
100000	$\min_{N_1} (1 - \eta_1 (N_1))$
100010	$\min_{N_1,N_3} (1 - \eta_1 (N_1) \eta_3 (N_3))$
000010	$\min_{N_3} (1 - \eta_3 (N_3))$
101010	$\min_{N_1,N_2,N_3} (1 - \eta_1 (N_1) \eta_2 (N_2) \eta_3 (N_3))$
001010	$\min_{N_2,N_3} (1 - \eta_2 (N_2) \eta_3 (N_3))$

The efficiency of the engine was calculated using the analytical model developed by Wagner and Papalambros [2]. The CVT transmission model compared was [3]. The battery SOC physics-based analytical model was adopted from [23], and the motor model used is a simple physics-based model, as described in [24]. The original three-level design and control optimization was published in [1], using the design variables shown in Figure 4.



Engine$_i$	$b_i, s_i, C_{r_i}, N_i, d_{i_i}, d_{e_i}$	N_1 N_2 N_3 N_4	P_{L-M} P_{M-H}
Motor/Gen$_i$	$K_{I_i}, K_i, Z_i, r_{a_i}, r_{c_i}$, $l_{a_i}, M_{a_i}, m_{ac_i}, R_{a_i}, r_{s_i}$		

FIGURE 4. Design, Control and Power Levels Optimized

For this study, the design of the engine and motor were fixed, and only the control was varied. The motors were assumed to be 50 kW DC motors; the parameters of the engine with two sets of cylinders for cylinder deactivation are given in Table 3. The parameter b is the bore of the engine cylinder, s is the stroke, C_r is the compression ratio, d_i is the intake valve diameter, and d_e is the exhaust valve diameter. For the motor, the parameter K_I is the integrator constant gain in the motor's lower level control, K_i is the torque constant, r_a is the armature radius, r_c is the conductor radius, l_a is the length of the armature, Z is the number of turns of the conductors in the armature, R_a is the armature resistance, r_s is the radius of the motor shaft, M_a is the mass of the armature, and m_{ac} is the mass of the armature conductor. Values of these parameters are given in [24].

TABLE 3. Engine Characteristics for Cylinder Deactivation

Variable	**Engine Fossil Fuel Cylinders**	**Engine Sustainable Fuel Cylinders**
b (mm)	70.9	71.4
s (mm)	101.3	100.0
C_r	10	16.8
d_i (mm)	24.5	22.6-23.2
d_e (mm)	21.3	19.3-19.7

The engine's operation was calculated based on a quasi-static assumption. In the development of the control algorithm, it was also assumed that the power switch points, P_{L-M} and P_{M-H}, were fixed at 40 kW and 80 kW, respectively. In future, this assumption could be relaxed, and the power switch points could be variable in order to provide additional degrees of freedom. The operation of the hybrid vehicle was simulated in Matlab, with the optimization performed by Matlab's SQP-based *fmincon* function. Optimal solutions were found, as indicated by satisfaction of the Karush-Kuhn-Tucker (KKT) conditions [25].

5 RESULTS

There are three aspects to the results of this hybrid vehicle's control, two general and one specific. The first aspect deals with the ability for the driver to immediately force himself or herself to limit their acceleration abilities in urban and suburban option 1 and 2. The vehicle was able to meet all the power requirements of the Japan 10-15 cycle for urban driving and NEDC for suburban 1 driving.

The second aspect is the the ability of the control algorithm to ensure that the motors and generators operate at optimal points for the vast majority of the driving cycle. In Figure 5, the full set of results is shown for all of the various driving cycles and driver options considered. It can be seen that that both the engines and the motors are able to operate at the desired points.

Furthermore, it can be seen in the graphs of the efficiencies that the motors and generators tend to be prioritized over engines by the control algorithm. This occurs because engines typically are significantly less efficient than motors, and the objective function is constructed such that a single inefficient component results in a low objective function value, representing an inefficient system. Consequently, while the objective function is based on a simple low-fidelity model of the system efficiency, it does reflect practical reality.

The third aspect of the results is the improvement in fuel economy resulting from this hybrid vehicle. Consider a single scenario, where Driver Option 3 (Suburban #2) is selected, which utilizes the drive cycle given in Table 1. For this design and a vehicle mass of 1800 kg, the fuel economy using cylinder deactivation alone is 14.1 mpg, with both sets of cylinders using gasoline. When Motor/Generator #1 is added into the optimization and all parameters are held constant, the fuel economy improves to 21.2 mpg. When the final element of this hybrid vehicle is added into the optimization, the use of an alternative fuel (in this case, hydrogen) in one set of cylinders, the optimal fuel economy further improves to 30.5 mpg. This clearly shows that this type of hybrid vehicle, with an increase in the number of degrees of freedom in the powertrain and the ability to incorporate the drivers knowledge of his or her intended route, can provide significant fuel economy benefits. However, running in EV, PEV1, and PEV2 allows the driver to benefit from plug-in capabilities while not sacrificing the HEV driving of long distances.

6 CONCLUDING REMARKS

In this paper, we have discussed the modes of operation and control challenges of a unique hybrid vehicle powertrain. This hybrid powertrain has 36 different modes of operation. Its controller utilizes both driver decisions, through four driver-selectable options, and information about the vehicle state to determine the optimal switching behavior between the different modes. The results of the optimization have shown that the hybrid can be effectively controlled to increase its operational efficiency. We fundamentally illustrate that a hybrid vehicle objective function can be changed from traditional bsfc and equivalent bsfc to our newly defined objective function. This objective function penalizes internal combustion engines over motor/generators. In addition, it penalizes multiple power sources over single sources.

There is a great amount of future work that should be done on this hybrid vehicle. This work includes the detailed design of the engine and surrounding powertrain, analysis of manufacturability, and further analysis and development of controls. Future work will also include the use of stochastic driving cycles or some form of predictive control, to determine performance and robustness under more realistic conditions. It may also include a combined optimization for design and control, in order to further increase the performance of the powertrain. The effect of an inappropriate driver option choice could also be studied, in order to determine what difficulties this might cause and to study how they could be alleviated by the control algorithm.

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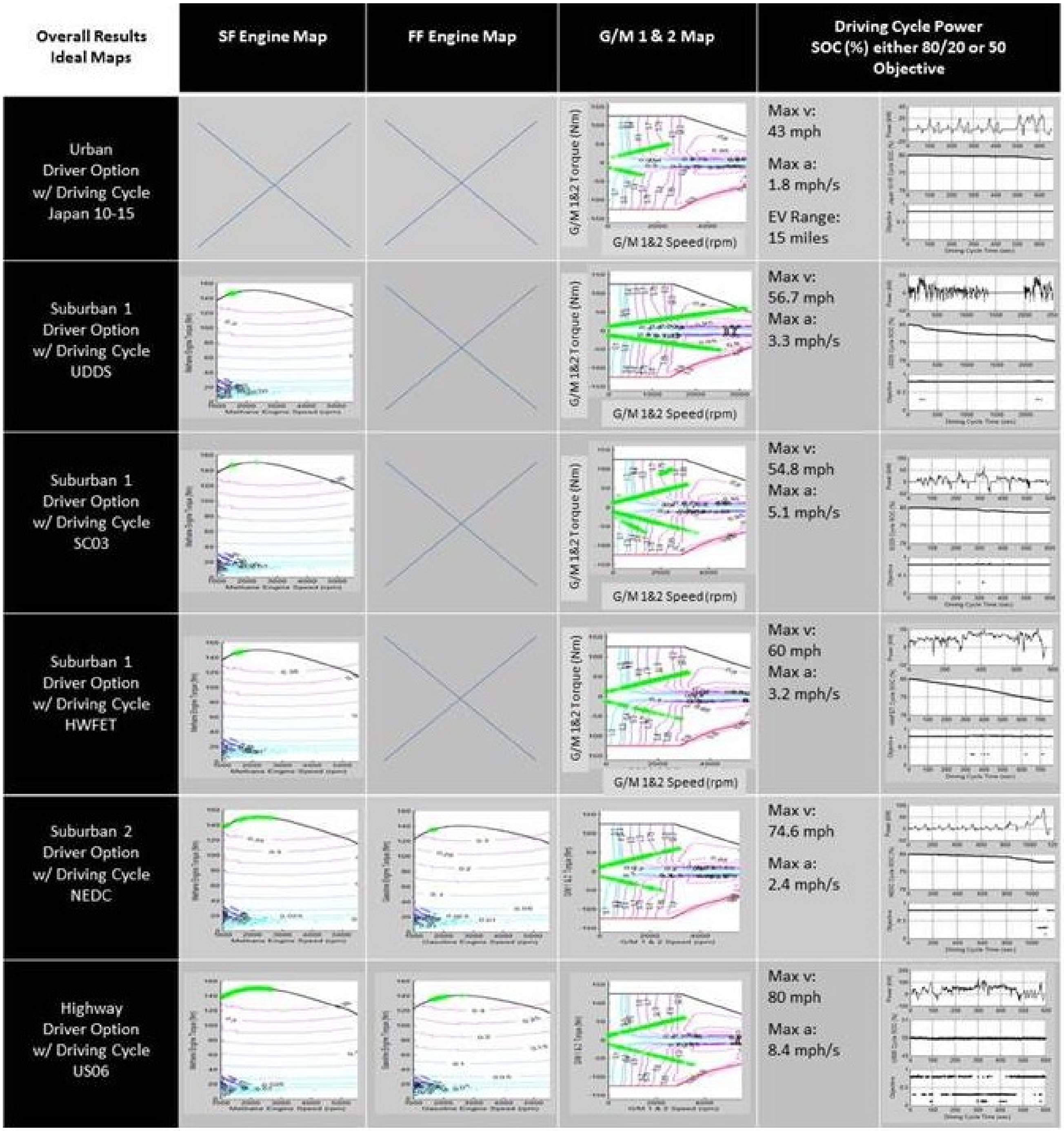

FIGURE 5. Results for 36 Mode Hybrid Vehicle Design

"Passive exhaust system with cylinder deactivation". In SAE World Congress. SAE Paper 2005-01-2351.

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