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DESIGN OF A CAM-ACTUATED ROBOTIC LEG

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ABSTRACT

This paper presents a concept for a single-degree-of-freedom robotic leg, where the lower and upper leg are each controlled by a cam. The two cams are mounted on a common shaft, and are rotating at the same speed. The relevant equations for the mechanism's kinematics are first developed, to express the position of the foot in terms of the cam's angular displacement and various design parameters such as link lengths. Next, the design problem is formulated as an optimization, where the objective is to minimize an error metric that compares the foot position to the desired trajectory of the foot. The constraints in the optimization problem include important parameters such as the pressure angle of the cams, as well as a set of constraints to ensure that the leg will fit on an appropriately sized legged robot. Finally, the results are discussed, with a focus on what the advantages and disadvantages of this leg design might be as compared to other types of robotic leg designs.

INTRODUCTION

While most common vehicles use wheels or tracks, robotic legs offer some advantages. They are more effective in rough terrain, do not require pavement or other road surfaces, and have a smaller environmental footprint [1]. Such legs may be designed with only one degree of freedom, or they may feature multiple degrees of freedom, with an associated control system.

Much of the recent work in the area of robotic legs has concentrated on multi-degree of freedom legs, particularly on their modeling and control, such as the five DOF robotic leg designed by Muscato and Spampinato [2, 3], the biarticulated robotic leg studied by Babič et al. [4], and a study of human leg function and its modeling [5]. Other work concentrates on the actuators themselves, e.g. [6–8]. The control literature also includes a wide variety of publications on the control of legged robots, e.g., [9–11]. Multi-degree of freedom legs, while they offer a great deal of flexibility, do require a more complex control scheme in order to take advantage of that flexibility.

A variety of simpler leg designs have been developed, which have less flexibility and which consequently require less intelligent control systems. Some examples of these include a leg using a pantograph mechanism [12], a leg using a Peaucellier mechanism [12], and a leg utilizing planetary gears [13]. The Peaucellier mechanism and the planetary gear leg both are single degree of freedom mechanisms; the pantograph requires two actuators. Each of these leg designs has its advantages and disadvantages, as detailed in the work describing each leg. Here, we present a leg design that utilizes cams, and perform an initial study of its feasibility.

PROBLEM FORMULATION

The robotic leg concept is a single degree of freedom mechanism, with two rigid links and two cams. Link 1 is attached to the robot with a revolute joint, with its motion controlled by cam 1. Link 2 is attached to link 1 with a revolute joint, and its motion is determined by the position of link 1 and cam 2. Both cams are on a common shaft, and are assumed to rotate at a constant angular velocity. A schematic of the mechanism, with variables identified, is given in Figure 1.

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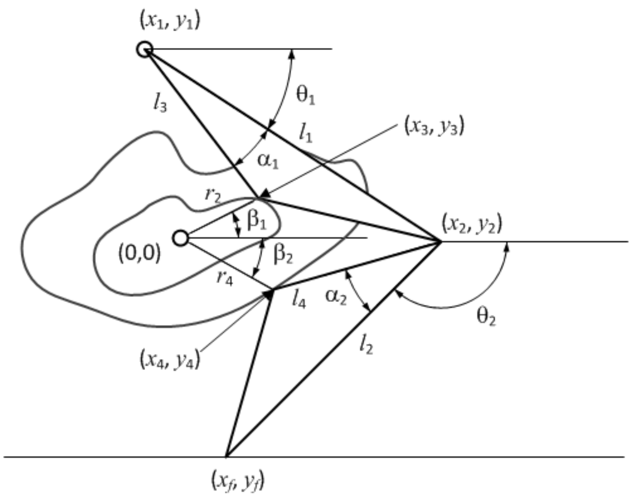

FIGURE 1. ROBOTIC LEG CONFIGURATION

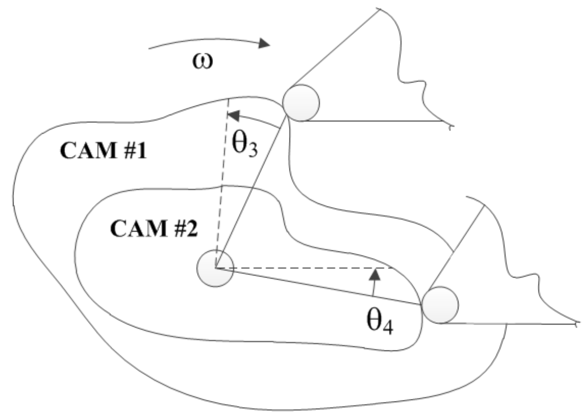

FIGURE 2. CAM ROTATIONS - DEFINED FROM INITIAL CONFIGURATION

Kinematics of Robotic Leg

Based on this configuration, the kinematic equations for the foot position were developed. The position of the foot, with respect to the fixed pivot point on link 1, is given by

$$x_f = x_1 + l_1 \cos\theta_1 + l_2 \cos\theta_2 \tag{1}$$
$$y_f = y_1 - l_1 \sin\theta_1 - l_2 \sin\theta_2 \tag{2}$$

The equations describing the kinematics of the leg are

$$x_1 + l_3 \cos(\theta_1 + \alpha_1) - r_2(\theta_3)\cos\beta_1 = 0 \tag{3}$$
$$y_1 - l_3 \sin(\theta_1 + \alpha_1) - r_2(\theta_3)\sin\beta_1 = 0 \tag{4}$$
$$x_1 + l_1 \cos\theta_1 - r_4(\theta_4)\cos\beta_2 + l_4 \cos(\theta_2 + \alpha_2) = 0 \tag{5}$$
$$y_1 - l_1 \sin\theta_1 + r_4(\theta_4)\sin\beta_2 - l_4 \sin(\theta_2 + \alpha_2) = 0 \tag{6}$$

where the angles θ_2 and θ_4 correspond to the location of the contact on the cam, as shown in Figure 2. This is related to the angles β_1 and β_2 by the relations

$$\theta_3 = \omega t + \beta_1 - \beta_{1_i} \tag{7}$$
$$\theta_4 = \omega t + \beta_2 - \beta_{2_i} \tag{8}$$

where β_{1_i} and β_{2_i} are the initial values of β_1 and β_2, respectively. For the purposes of initial design, we assume that the changes in β_1 and β_2 are negligible. Therefore, $\theta_3 = \theta_4 = \omega t$.

Path of Foot

The desired path of the foot is a closed, piecewise continuous function. In the first part of the path, when the foot is in contact with the ground, it is to follow a straight-line path. In the second part of the path, the foot is to follow a curved path, where it is lifted off of the ground and moved forward. At the end of this motion, the foot should be in contact with the ground at the start of the straight-line path. The desired path is shown in Figure 3, with its mathematical expression given by Eqs. (9) - (12), where x_{f_i}, y_{f_i} represent the initial position of the foot, l_5 represents the full horizontal stroke of the foot, and l_6 represents the highest position of the foot as it returns to its initial position. The full motion of the foot corresponds to a complete rotation of the cams, and so the x and y coordinates of the foot are parameterized based on cam rotations, and are piecewise continuous over two intervals representing the contact of the foot with the ground, covering the range of $0 \le \theta_3 < \frac{3\pi}{2}$, and the return of the foot when it is out of contact with the ground, covering the range of $\frac{3\pi}{2} \le \theta_3 < 2\pi$. During the contact phase, the foot position is given by

$$x_f = x_{f_i} - \frac{2l_5}{3\pi}\theta_3 \tag{9}$$
$$y_f = y_{f_i} \tag{10}$$

and during the non-contact phase of the motion, the foot position is given by

$$x_f = x_{f_i} + l_5\left(\frac{2}{\pi}\theta_3 - 4\right) \tag{11}$$
$$y_f = y_{f_i} - \frac{16l_6}{\pi^2}\theta_3^2 + \frac{56l_6}{\pi}\theta_3 - 48l_6 \tag{12}$$

The lift height of the foot, l_6, is chosen in order to raise the foot high enough to clear small obstacles, but kept low enough to

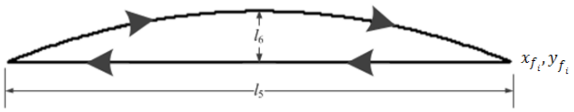

FIGURE 3. DESIRED PATH OF FOOT

TABLE 1. VALUES OF LEG PARAMETERS

Parameter	Value	Units
x_1	0.000	m
y_1	0.500	m
x_{f_1}	0.250	m
y_{f_2}	-1.500	m
l_1	0.875	m
l_2	1.500	m
l_3	0.250	m
l_4	0.500	m
l_5	0.500	m
l_6	0.125	m
α_1	$\frac{\pi}{6}$	rad
α_2	$\frac{\pi}{36}$	rad

minimize the energy requirement to lift it. The stroke of the foot, l_5, is chosen to allow the robot to move forward a reasonably large distance, proportional to the scale of the robot, during each cycle. The values selected for these parameters are given in Table 1, with other parameters used in the design.

Optimization Problem Formulation

The design of the leg was performed through a two-step process. In the first step, all of the parameters describing the leg except for the cam profile were specified, as given in Table 1, and the cam dimensions were calculated for 72 evenly spaced points on the cams. This step yielded cam profiles that would produce the desired motion, as shown in Figure 4; however, the cams displayed features such as sharp points that are undesirable [14].In order to deal with this issue, this design was used as the starting point in an optimization, in which some of the parameters for the leg design and the cam dimensions were variables.

This optimization problem was designed to minimize the error between the actual position of the foot and its desired position, subject to a set of constraints. These constraints included constraints on the link lengths to ensure kinematic compatibility

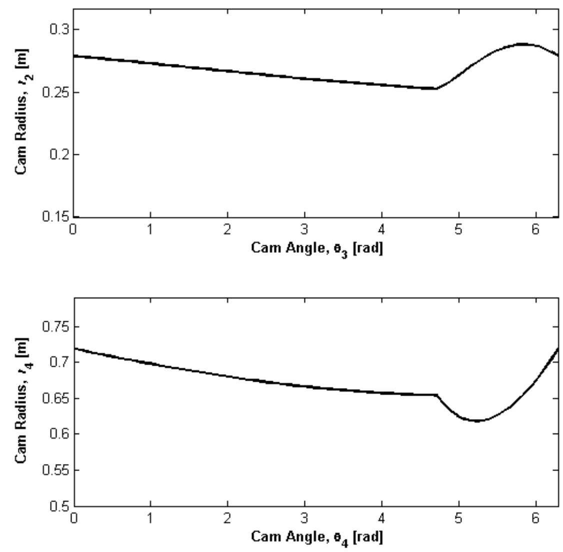

FIGURE 4. PRELIMINARY CAM DESIGN - PRIOR TO OPTIMIZATION

and a set of constraints on the cam profile. The cam profile constraints were formulated to ensure that the cam profile is closed, and that the profile is not too steep. The parameters that were fixed in this optimization were x_1, y_1, x_{f_1}, y_{f_2}, l_1, and l_2, with the values given in Table 1, and the link lengths and angles l_3, l_4, α_1 and α_2 were selected as variables, in addition to the cam profiles. The full optimization problem is described by Eqs. (13) - (21).

$$\min_{\mathbf{r_2},\mathbf{r_4},\mathbf{l_3},\mathbf{l_4},\alpha_1,\alpha_2} \sum_{i=0}^{72} \left(\left(x_{f_a} - x_{f_d}\right)^2 + \left(y_{f_a} - y_{f_d}\right)^2 \right) \tag{13}$$

$$\text{subject to} \quad x_1^2 + y_1^2 - r_{2_i}^2 - l_3^2 \leq 0 \forall i = 1,\ldots,n \tag{14}$$

$$x_2^2 + y_2^2 - r_{4_i}^2 - l_4^2 \leq 0 \forall i = 1,\ldots,n \tag{15}$$

$$r_{2_1} - r_{2_n} = 0 \tag{16}$$

$$r_{4_1} - r_{4_n} = 0 \tag{17}$$

$$0 < r_{2_i} \leq r_{2_{max}} \forall i = 1,\ldots,n \tag{18}$$

$$0 < r_{4_i} \leq r_{4_{max}} \forall i = 1,\ldots,n \tag{19}$$

$$\left| \frac{\Delta r_{2_i}}{\Delta \theta} \right| \leq \Delta r_{2_{max}} \forall i = 1,\ldots,n \tag{20}$$

$$\left| \frac{\Delta r_{4_i}}{\Delta \theta} \right| \leq \Delta r_{4_{max}} \forall i = 1,\ldots,n \tag{21}$$

TABLE 2. VALUES OF LEG PARAMETERS

Parameter	Value	Units
l_3	0.500	m
l_4	0.500	m
α_1	0.175	rad
α_2	2.793	rad

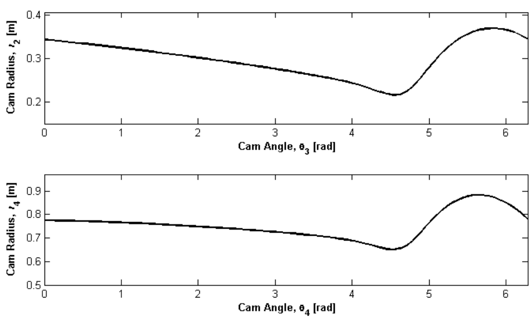

FIGURE 5. CAM PROFILES AFTER OPTIMIZATION

RESULTS

The optimization problem was solved using Matlab's *fmincon* function. This function is based on Sequential Quadratic Programming (SQP), a commonly used algorithm for optimization of non-linear objective functions with non-linear inequality and/or equality constraints [15]. Due to the highly non-linear nature of the problem, and the possibility of unreachable configurations, the implementation of this optimization was not very robust; however, it was able to converge to a design that produced cams with a smoother transition between the two phases of the motion. Values for the variables l_3, l_4, α_1, and α_2 are given in Table 2, and the cam profiles are given in Figure 5.

A drawing of the final leg configuration is given in Figure 6. A comparison of the actual path of the foot and the desired foot path is given in Figure 7, and the error between the desired and actual path of the foot is given in Figure 8.

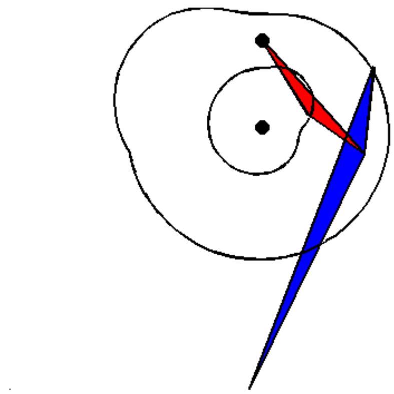

FIGURE 6. FINAL LEG DESIGN

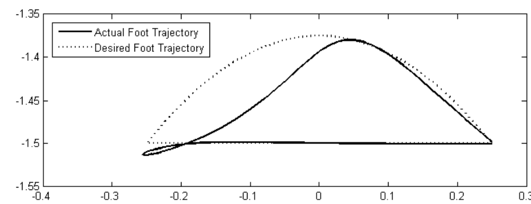

FIGURE 7. COMPARISON OF DESIRED AND ACTUAL FOOT TRAJECTORY

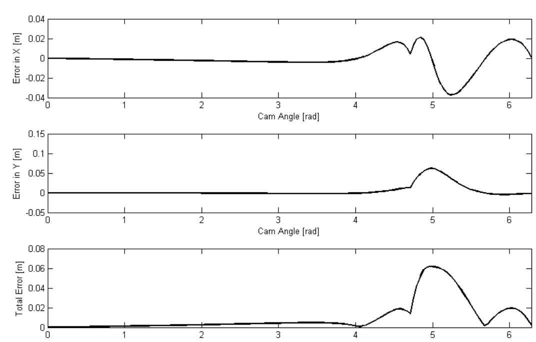

FIGURE 8. ERRORS IN FOOT TRAJECTORY

DISCUSSION

It can be seen that the foot deviated somewhat from its desired path during the return phase of the motion. This deviation from the prescribed path does not impair the ability of the leg to perform its function, as the leg is not in contact with the ground during this phase of the motion. It can also be noted that the foot deviates from its desired path at the end of the ground contact portion of its motion, prior to being lifted off of the ground. While this is not desirable and will produce some unwanted disturbance to the body's motion, it is quite a small vertical disturbance, less than 2 cm, and is therefore judged to be acceptable. Based on the simple model used here, and the small set of criteria used in the design, this leg design is feasible. With refinement to the equations and a more robust design method, further improvements in the design should be possible.

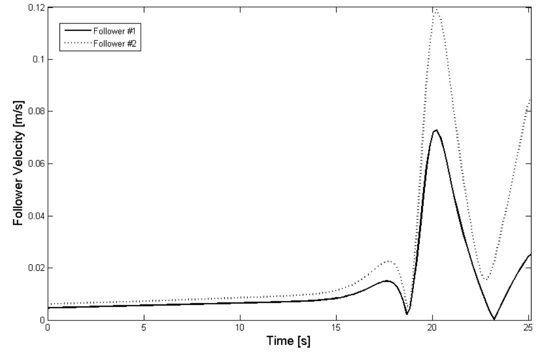

FIGURE 9. VELOCITIES OF CAM FOLLOWERS

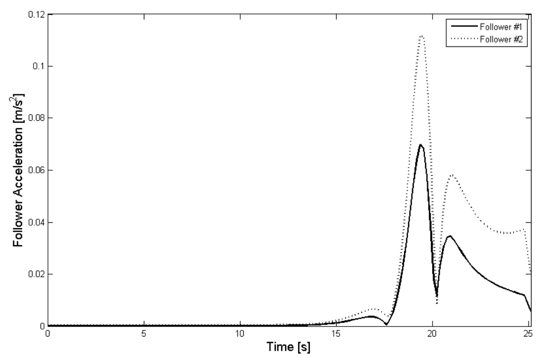

FIGURE 10. ACCELERATIONS OF CAM FOLLOWERS

One way in which the design could be enhanced is by selecting an appropriate function to describe the cam, and then optimizing it for a small number of parameters that control the function. This would reduce the number of design variables considerably, which would make the problem more tractable, and would provide the opportunity to derive additional constraints relevant to the design of appropriate cams. A preliminary attempt was made to do this, by fitting a single equation to the points found for this cam, and it was found that each cam profile could be described, to a high degree of accuracy, by using two quadratic equations and a continuous approximation to the Heaviside step function. In this way, Cam 1 could be described by the equation

$$\begin{aligned} r_2(\theta_3) = &\left(-0.0016\theta_3^2 - 0.0177\theta_3 + 0.3434\right) \\ &\left(1 - \frac{1}{1+e^{-6(\theta_3-1.5\pi)}}\right) \\ &+ \left(-0.1181\theta_3^2 + 1.3753\theta_3 - 3.6342\right) \\ &\left(\frac{1}{1+e^{-6(\theta_3-1.5\pi)}}\right) \end{aligned} \tag{22}$$

Similarly, Cam 2 could be described by

$$\begin{aligned} r_4(\theta_4) = &\left(-0.0035\theta_4^2 - 0.0062\theta_4 + 0.7758\right) \\ &\left(1 - \frac{1}{1+e^{-6(\theta_4-1.5\pi)}}\right) \\ &+ \left(-0.2530\theta_4^2 + 2.8518\theta_4 - 7.1518\right) \\ &\left(\frac{1}{1+e^{-6(\theta_4-1.5\pi)}}\right) \end{aligned} \tag{23}$$

If these types of equations are used to represent the cams, then six variables suffice to describe each cam, regardless of how many points are considered. This would reduce the size of the optimization problem significantly, and may make the problem more tractable.

This change to the problem also allows changes in β_1 and β_2 to be more easily accommodated, eliminating the need for the assumptions that $\beta_1 = \beta_{1_i}$ and $\beta_2 = \beta_{2_i}$, and allows for more efficient and accurate calculations of velocity, acceleration, and jerk. While the optimization was not carried out using these equations for the cam profile, they were implemented to calculate the velocity, acceleration, and jerk of each cam follower, assuming a rotational speed of $\omega = 0.25$rad/s for both cams. Results of these calculations are shown in Figures 9 - 11. It can be seen that the velocity, acceleration, and jerk are all small for the majority of the motion, while the foot is in contact with the ground. They increase during the lift-off and return phase, although they are still relatively small. Since this phase of the motion is less critical than the phase where the leg is in contact with the ground, it should be possible to decrease the maximum values seen, if necessary, without impairing functionality of the leg.

CONCLUSION

As shown by this work, a single degree of freedom robotic leg utilizing cams to actuator a simple linkage is feasible. However, the design of suitable cams can be challenging, since the relevant design equations are highly nonlinear and the foot motions in the horizontal and vertical directions are coupled. In addition, while one of the issues initially present with the cams, the sharp corner as the foot was lifted, has been addressed, cam design is a highly complex subject and it is possible that there are additional issues that need to be addressed in order to produce feasible cams.

The question of efficiency is an important one which has not been addressed in this work; the performance of this leg, compared to other simple leg designs such as the Peaucellier mechanism, pantograph, and planetary gear leg, has not been studied. Such an evaluation will be the subject of future work.

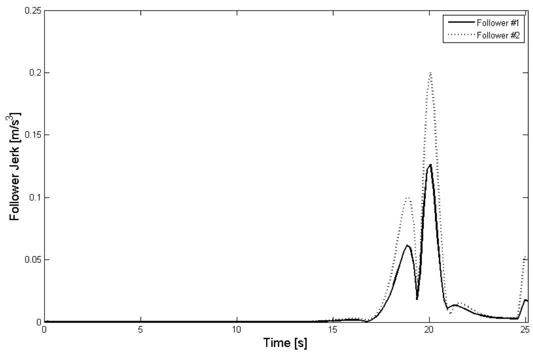

FIGURE 11. JERK OF CAM FOLLOWERS

If such an evaluation shows that this leg design is efficient as well as feasible, then it could be considered for simple walking robots. As with all single degree of freedom mechanisms, once designed its output stroke cannot be changed without changing the mechanism, and in many applications a more flexible leg with greater degrees of freedom, such as that described in [2], would be appropriate. However, in some cases a simpler mechanism without complex control will be suitable, and a leg such as this one may be a desirable solution. Further work will show whether this is the case.

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